va00aa

briefly

[called by: .] [calls: .]

contents

1.1 representation of scalar potential

1. The (stellarator-symmetric) scalar potential may be written

$$\Phi = I\,\theta + G\,\zeta + \sum_{l,i} \phi_{li}\, T_l\, \sin\alpha_i. \tag{1}$$

where the enclosed toroidal and poloidal currents, I and G, are assumed to be given, the ϕ_{li} are to be determined, the $T_l(s)$ are the Chebyshev polynomials, and $\alpha_i \equiv m_i\theta - n_i\zeta$.

2. The magnetic field, $\mathbf{B} \equiv \nabla\Phi$, is

$$\nabla\Phi = I\,\nabla\theta + G\,\nabla\zeta + \phi_{li}\,\left(T_l'\,\sin\alpha_i\nabla s + m_i T_l\,\cos\alpha_i\nabla\theta - n_i T_l\,\cos\alpha_i\nabla\zeta\right). \tag{2}$$

3. The current, $\mathbf{j} \equiv \nabla\times\mathbf{B} = \nabla\times\nabla\Phi$, is identically zero by construction.

1.2 constrained minimization

1. Introduce the functional

$$F \equiv \frac{1}{2}\int_{\mathcal{V}} \nabla\Phi\cdot\nabla\Phi\, dv + \int_{\partial V} \lambda\,(\nabla\Phi - \mathbf{B})\cdot d\mathbf{s} \tag{3}$$

where $\lambda(\theta,\zeta) \equiv \sum_i \lambda_i \sin\alpha_i$ is a Lagrange multiplier used to enforce the boundary condition that $\sqrt{g}\,\nabla\Phi\cdot\nabla s = \sqrt{g}\,\mathbf{B}\cdot\nabla s$, where the Jacobian-weighted, normal component of the *total* magnetic field, $\sqrt{g}\,\mathbf{B}\cdot\nabla s$, is assumed to be given on $\partial\mathcal{V}$.

2. The independent degrees-of-freedom in the solution (which, for coding reasons, must be 'packed' into a single vector) are $\mathbf{a} \equiv \{\phi_{li}, \lambda_i\}$.

3. The enclosed currents, $\boldsymbol{\psi} \equiv (I, G)^T$, produce 'inhomogeneous terms' that drive non-trivial solutions.

1.3 first derivatives

1. The first derivatives of F with respect to the independent degrees-of-freedom are:

$$\begin{aligned}
\frac{\partial F}{\partial \phi_{li}} &\equiv \int_{\mathcal{V}} (T_l' \sin\alpha_i\,\nabla s + m_i T_l \cos\alpha_i\,\nabla\theta - n_i T_l \cos\alpha_i\,\nabla\zeta)\cdot\nabla\Phi\, dv \\
&+ \int_{\partial\mathcal{V}} \lambda\,(T_l' \sin\alpha_i\,\nabla s + m_i T_l \cos\alpha_i\,\nabla\theta - n_i T_l \cos\alpha_i\,\nabla\zeta)\cdot d\mathbf{s} \qquad (4)\\
\frac{\partial F}{\partial \lambda_i} &\equiv \int_{\partial V} \sin\alpha_i\,(\nabla\Phi - \mathbf{B})\cdot d\mathbf{s} \qquad (5)
\end{aligned}$$

1.4 second derivatives

1. The second derivatives, which constitute the (symmetric) matrix $\mathcal{A}$, are:

$$
\begin{array}{rlccccccccc}
\displaystyle\frac{\partial}{\partial \phi_{pj}} \frac{\partial F}{\partial \phi_{li}} & = & & & & & & & & & \\
& + & & & \int ds & T_l' & T_p' & \oint\!\!\oint d\theta d\zeta & \sin\alpha_i & \sin\alpha_j & \bar{g}^{ss} \\
& + & & m_j & \int ds & T_l' & T_p & \oint\!\!\oint d\theta d\zeta & \sin\alpha_i & \cos\alpha_j & \bar{g}^{s\theta} \\
& + & m_i & & \int ds & T_l & T_p' & \oint\!\!\oint d\theta d\zeta & \cos\alpha_i & \sin\alpha_j & \bar{g}^{s\theta} \\
& - & & n_j & \int ds & T_l' & T_p & \oint\!\!\oint d\theta d\zeta & \sin\alpha_i & \cos\alpha_j & \bar{g}^{s\zeta} \\
& - & n_i & & \int ds & T_l & T_p' & \oint\!\!\oint d\theta d\zeta & \cos\alpha_i & \sin\alpha_j & \bar{g}^{s\zeta} \\
& + & m_i & m_j & \int ds & T_l & T_p & \oint\!\!\oint d\theta d\zeta & \cos\alpha_i & \cos\alpha_j & \bar{g}^{\theta\theta} \\
& - & m_i & n_j & \int ds & T_l & T_p & \oint\!\!\oint d\theta d\zeta & \cos\alpha_i & \cos\alpha_j & \bar{g}^{\theta\zeta} \\
& - & n_i & m_j & \int ds & T_l & T_p & \oint\!\!\oint d\theta d\zeta & \cos\alpha_i & \cos\alpha_j & \bar{g}^{\theta\zeta} \\
& + & n_i & n_j & \int ds & T_l & T_p & \oint\!\!\oint d\theta d\zeta & \cos\alpha_i & \cos\alpha_j & \bar{g}^{\zeta\zeta},
\end{array} \tag{6}
$$

$$
\begin{array}{rlcccccc}
\displaystyle\frac{\partial}{\partial \lambda_{j}} \frac{\partial F}{\partial \phi_{li}} & = & & & & & & \\
& + & & T_l' & \oint\!\!\oint d\theta d\zeta & \sin\alpha_j & \sin\alpha_i & \bar{g}^{ss} \\
& + & m_i & T_l & \oint\!\!\oint d\theta d\zeta & \sin\alpha_j & \cos\alpha_i & \bar{g}^{s\theta} \\
& - & n_i & T_l & \oint\!\!\oint d\theta d\zeta & \sin\alpha_j & \cos\alpha_i & \bar{g}^{s\zeta},
\end{array} \tag{7}
$$

$$
\begin{array}{rlcccccc}
\displaystyle\frac{\partial}{\partial \phi_{pj}} \frac{\partial F}{\partial \lambda_{i}} & = & & & & & & \\
& + & & T_p' & \oint\!\!\oint d\theta d\zeta & \sin\alpha_i & \sin\alpha_j & \bar{g}^{ss} \\
& + & m_j & T_p & \oint\!\!\oint d\theta d\zeta & \sin\alpha_i & \cos\alpha_j & \bar{g}^{s\theta} \\
& - & n_j & T_p & \oint\!\!\oint d\theta d\zeta & \sin\alpha_i & \cos\alpha_j & \bar{g}^{s\zeta},
\end{array} \tag{8}
$$

$$
\frac{\partial}{\partial \lambda_{j}} \frac{\partial F}{\partial \lambda_{i}} = 0, \tag{9}
$$

where $\bar{g}^{\mu\nu} \equiv \sqrt{g}\, g^{\mu\nu}$.

2. The required integral information is provided in `TToo`, `TToe`, `TTeo` and `TTee`, which are calculated in ma00aa; and in `Too`, `Toe`, `Teo` and `Tee`, which are also calculated in ma00aa.

1.5 inhomogeneous terms

1. The inhomogeneous terms, which constitute the 'right-hand-side' matrix $\mathcal{B}$, are

$$
\begin{array}{rlcccccc}
\mathcal{D}_{li} & = & & & & & & \\
& + & I & & \int ds & T_l' & \oint\!\!\oint d\theta d\zeta & \sin\alpha_i & \bar{g}^{s\theta} \\
& + & I & m_i & \int ds & T_l & \oint\!\!\oint d\theta d\zeta & \cos\alpha_i & \bar{g}^{\theta\theta} \\
& - & I & n_i & \int ds & T_l & \oint\!\!\oint d\theta d\zeta & \cos\alpha_i & \bar{g}^{\theta\zeta} \\
& + & G & & \int ds & T_l' & \oint\!\!\oint d\theta d\zeta & \sin\alpha_i & \bar{g}^{s\zeta} \\
& + & G & m_i & \int ds & T_l & \oint\!\!\oint d\theta d\zeta & \cos\alpha_i & \bar{g}^{\theta\zeta} \\
& - & G & n_i & \int ds & T_l & \oint\!\!\oint d\theta d\zeta & \cos\alpha_i & \bar{g}^{\zeta\zeta}
\end{array} \tag{10}
$$

$$
\begin{array}{rlcccc}
D_{i} & = & & & & \\
& + & I & \oint\!\!\oint d\theta d\zeta & \sin\alpha_i & \bar{g}^{s\theta} \\
& + & G & \oint\!\!\oint d\theta d\zeta & \sin\alpha_i & \bar{g}^{s\zeta} \\
& - & & \oint\!\!\oint d\theta d\zeta & \sin\alpha_i & b
\end{array} \tag{11}
$$

where $b \equiv \sqrt{g}\,\mathbf{B}\cdot\nabla s$ on the computational boundary.

1.6 boundary conditions

1. On the inner boundary, $s = -1$, which is the plasma boundary, the normal component of the total magnetic field is zero.

2. On the outer boundary, $s = +1$, which is the 'computational' boundary, the normal component of the total magnetic field, $\mathbf{B}_T \equiv \mathbf{B}_C + \mathbf{B}_P$, must be provided.

3. Usually, only the magnetic field produced by the external currents, $\mathbf{B}_C$, is known a-priori. The magnetic field produced by plasma currents, $\mathbf{B}_P$, must be determined iteratively as part of the free-boundary equilibrium calculation (see vc00aa, bn00aa and xspech).

1.7 dependencies

1. The required integrals over the Chebyshev polynomials and the metric elements are provided in `TTee`, `TTeo`, `TToe` and `TToo`; `Tee`, `Teo`, `Toe` and `Too`; and `Te` and `To`; all of which are allocated in fc02aa, and defined and calculated in ma00aa.

2. Additionally, the mode identification arrays, `ki`, `guvmne`, and `guvmno` are required. These are described in al00aa.